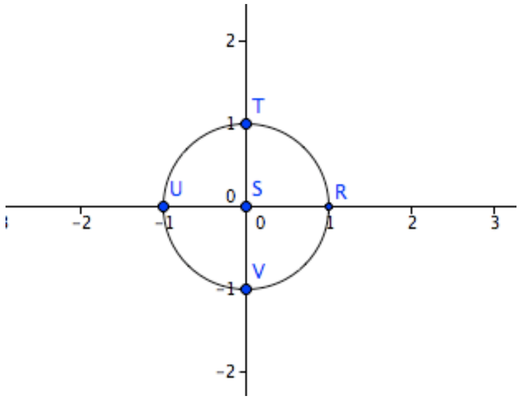
Problem set 2.2.1 Number 6 from *Mathematics for High School Teachers* (pg. 53 - 54)

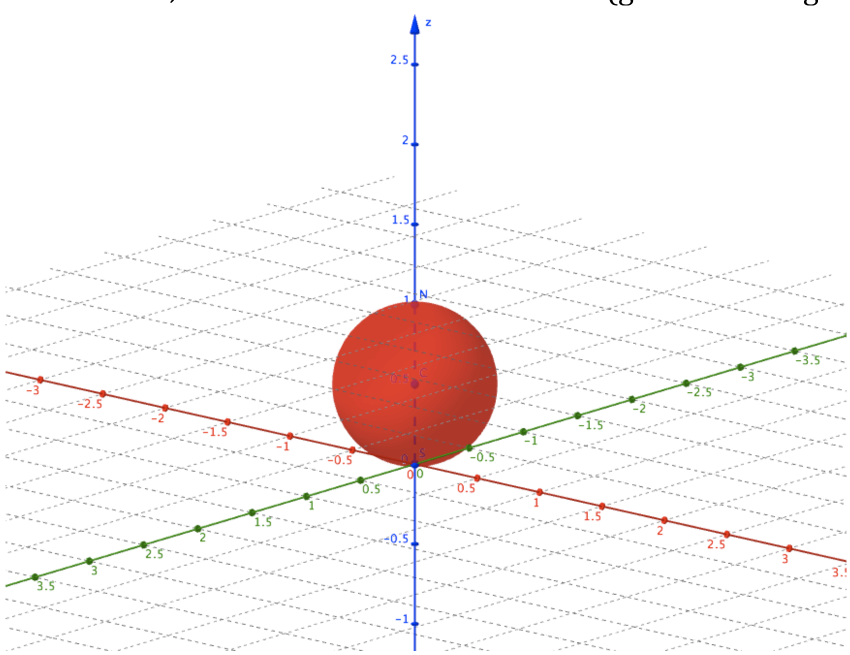
"Describe precisely the set on the Riemann sphere that corresponds to the indicated set in the complex plane."

a. "the unit circle $\{z \in C: |z| = 1\}$

The absolute value of z would be all points on the complex plane that form a circle at the origin S with a radius of 1 (given in the figure below) where the imaginary axis is the vertical axis and the real axis is the horizontal axis.



A Riemann sphere is depicted in the three-dimensional plane as a sphere with the diameter of 1, where we let point N (which we will call the north pole on the sphere) be located at (0,0,1), and point S (which we will call the south pole on the sphere) be located at the origin. The green axis is our imaginary axis, the red axis is our real number axis, and the blue axis is the z – axis.(given in the figure below).



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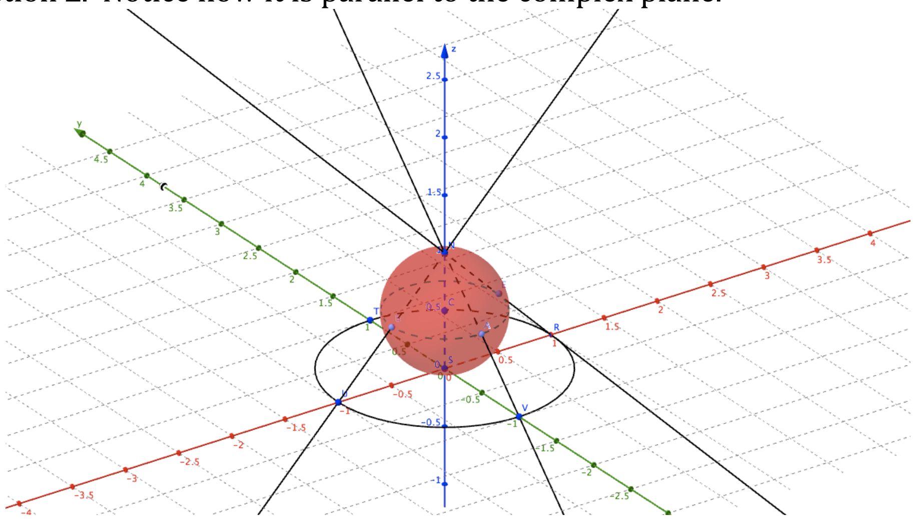
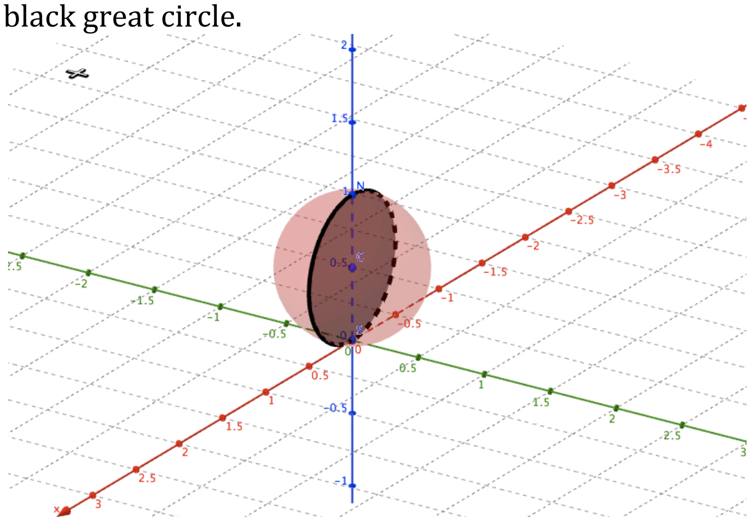
To get a stereographic projection we connect all points z on the complex plane with the north pole N on our Riemann sphere. The intersection of these lines with the sphere creates a set of points we will call Z on the Riemann sphere. In the pictures below created by Geogebra there are three of these lines that intersect the sphere. We could draw an infinite number of them. The dotted great circle in the picture is our Stereographic projection Z. Notice how it is parallel to the complex plane.



b. the real axis {z ∈ C: Im(z) = 0}

Jumping right into this problem we have to imagine a plane p that extends the real (red) axis along the z-axis. Where this intersects our Riemann sphere is our stereographic projection Z. Sherry West thought of it like a quarter perched on edge on the real axis. In the picture below it will be the black great circle.



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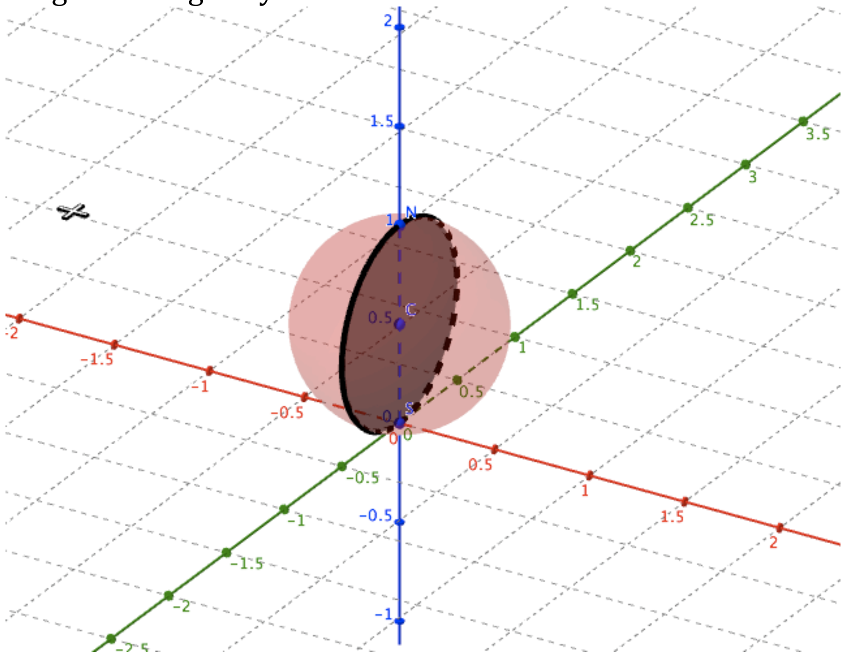
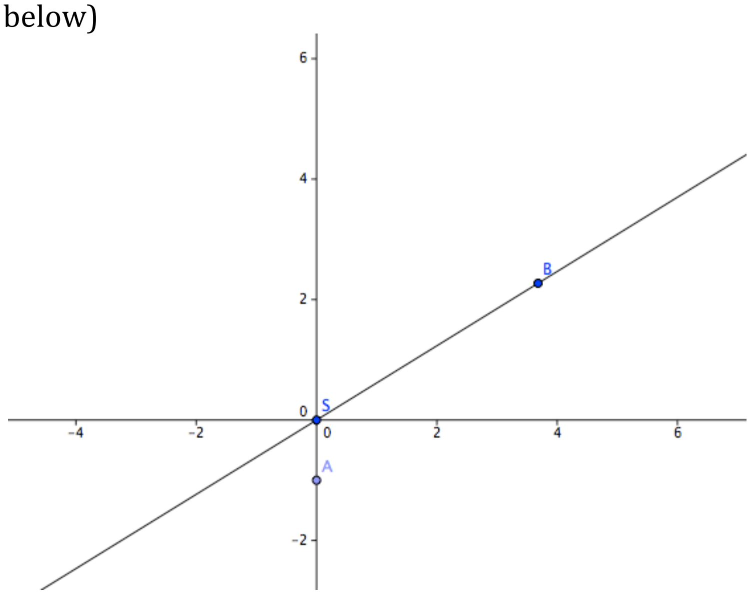
c. The imaginary axis {z ∈ C: Re(z) = 0}
This projection is very similar to problem b but the coin will lie on edge along the green imaginary axis.



d. A straight line in the complex plane that passes through the origin.
In this projection I represent Z by first placing a line in line that passes through the origin of the two dimensional plane (as represented by the plane below)



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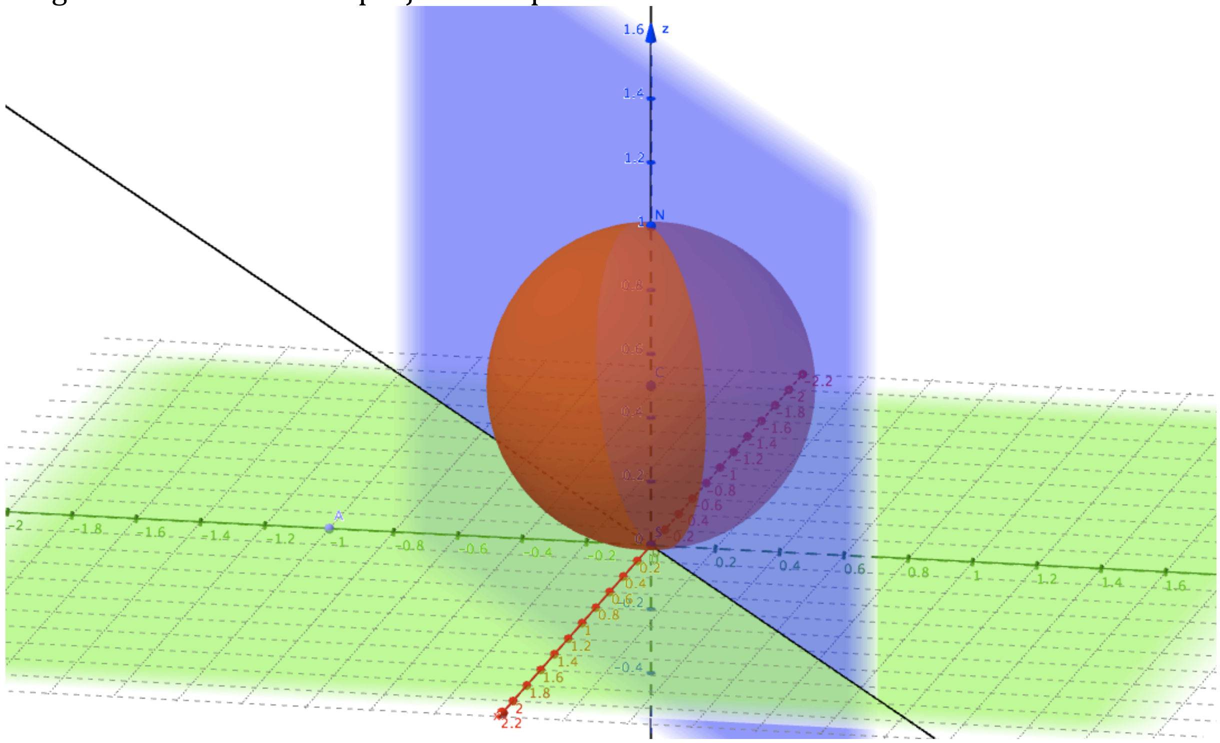
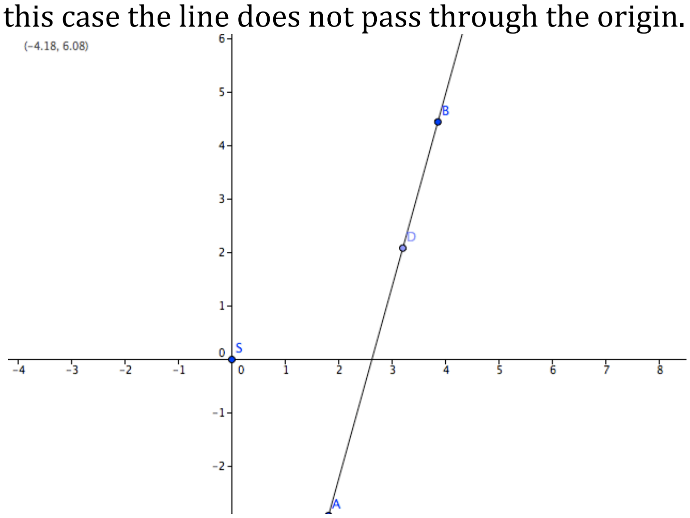
Extending this line perpendicularly to the imaginary plane in green, I form plane b (the blue plane). The intersection of this plane and the sphere forms the great circle that is our projection Z pictured below.



e. A straight line in the complex plane that does not pass through the origin. Similarly I will construct a line that lies on the two dimensional complex plane as below. In this case the line does not pass through the origin.



Now I can construct a plane that makes point N and the line I constructed in the complex plane coplanar. Where this plane intersects the sphere is our small circle Z. Below is a few different views of this situation with lines in different orientations and different views.

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